Received: *July 4, 1998*, Accepted: *July 15, 1998*

Self-duality and new TQFTs for forms

Laurent Baulieu

CERN, Geneva, Switzerland
LPTHE, Paris, France.*
E-mail: laurent.baulieu@cern.ch

Eliezer Rabinovici†

CERN, Geneva, Switzerland
Racah Institute of Physics, Hebrew University, Jerusalem, Israel.
E-mail: eliezer.rabinovici@cern.ch

Abstract: We discuss theories containing higher-order forms in various dimensions. We explain how Chern–Simons-type theories of forms can be defined from TQFTs in one less dimension. We also exhibit new TQFTs with interacting Yang–Mills fields and higher–order forms. They are obtained by the dimensional reduction of TQFTs whose gauge functions are free self-duality equations. Interactions are due to the gauging of global internal symmetries after dimensional reduction. We list possible symmetries and give a brief discussion on the possible relation of such systems to interacting field theories.

Keywords: Topological Field Theories, Chern-Simons Theories, Field Theories in Higher Dimensions.

JHEP06(1998)006

---

*UMR CNRS associée aux Universités Pierre et Marie Curie (Paris VI) et Denis Diderot (Paris VII).

†Work supported in part by the Israel Academy of Sciences and Humanities, Center of Excellence Program and the American–Israeli Binational Science Foundation-BSF.

Contents

1. Introduction

Recent interesting results shed new light on field theories in dimensions higher than four which contain no explicit graviton interactions and involve higher–order forms. Motivated by a new understanding of non perturbative features of string theory and insights obtained for an M theory, a rich structure of supersymmetric theories was uncovered in five and six dimensions [1]. Another set of results concerns the exploration of the holography principle. According to that principle the degrees of freedom of some higher-dimensional systems are actually fully accounted for by degrees of freedom residing only on the boundary of those systems [2]. It has indeed been suggested [3] that p-form gauge fields with $p > 1$ could be used to implement these ideas.

In this paper we will study theories of forms in an attempt to enrich the class of models depending on such fields. Our starting point is Topological Field Theory (TQFT). TQFTs are quantum field theories that possess observables depending only on global degrees of freedom. Examples are the cases of the four and eight dimensional topological Yang–Mills theory [4][5][6]. In these cases, the topological behaviour is particularly clear because the TQFT actions can be described as BRST-invariant gauge-fixings of classical topological invariants. Moreover, they have the interesting feature that they are linked by twist to supersymmetric theories with physical particles. For those of the higher-dimensional TQFTs which can be untwisted into theories with Poincaré supersymmetry, one could hope that, the latter could inherit from the former properties which allow them to pass the barrier of naive non renormalizability.

It must be noted that the TQFTs depending on forms with $p > 1$ are likely to have a rich content because, at least in the abelian case, it is known that a p-form can be defined with a status analogous to that of a connection, so that one expects non vanishing topological invariants when one integrates exterior products of their curvatures over certain spaces (see e.g. [7]).

This work has two aspects. The first part aims at finding new Chern-Simons (CS) like theories, and defining them. What we mean by a classical CS-type action is an action made of exterior products of p-forms and their curvatures (the Yang–Mills Chern–Simons actions are particular cases). It is independent of the metric and of first order and may have gauge invariances for forms. The Hamiltonian vanishes modulo gauge transformations. It was suggested that CS theories embody the holography principle. We show that the procedure initiated in [8] gives a method for defining such theories. Given a TQFT in D dimensions, based on the BRST quantization of a topological term, we observe that it can often be put in correspondence with a CS-type theory in $(D+1)$ dimensions. The way we define the CS theory implies that it is invariant under a symmetry operation Q, such that

$$Q\varphi = \Psi \tag{1.1}$$

and that there is a mapping between the fields of the TQFT in D dimensions and those of the CS theory in $(D+1)$ dimensions.

The CS action decomposes into a Q-invariant but not Q-exact part and a Q-exact part. The Q-exact part can be seen as a topological gauge-fixing term, analogous to that occurring in the D-dimensional theory. Moreover, the topological gauge functions in $(D+1)$ dimensions are inspired by those of the TQFT in D dimensions.[1] Their role is to provide in a Q-exact way terms such that the complete action is second order for the bosons and first order for the fermions. The Q-invariant but not Q-exact part contains a classical CS term, plus supersymmetric terms. It exists because the cohomology with ghost number zero of Q is not empty in $(D+1)$ dimensions, contrary to what occurs in D dimensions.

The symmetry (1.1) indicates a gauge invariance of the topological type. Its existence simplifies the counting of local degrees of freedom of the CS action: by establishing a pairing between the bosons φ and fermions Ψ, it allows an investigation of the topological properties of the theory, at least formally. As a matter of fact, the supersymmetric formalism for the CS-type theory in $(D+1)$ dimensions establishes almost by definition that the theory has no gauge-invariant local degrees of freedom in the "bulk", since only global excitations can remain in the Hilbert space defined by the principle of Q-invariance. This agrees with the property that the classical Hamiltonian vanishes (modulo gauge transformations). In the absence of such a symmetry, the counting of degrees of freedom for TQFTs based on CS first-order actions is more involved, except in the three -dimensional Chern–Simons Yang–Mills case where one can directly prove that the theory does indeed describe only global degrees of freedom

[1] The topological gauge functions are gauge-invariant under the ordinary gauge symmetries of forms: they determine a gauge invariant Lagrangian whose residual ordinary gauge-invariance must be gauge-fixed, eventually by the standard methods.

(in particular by solving Gauss's law). Other gauges exist, which can be enforced in a Q-invariant way, such that all fermions decouple, and the action formally reduces to a classical bosonic CS action, whose direct quantization is challenging.

We can tentatively give a physical interpretation to this quite general mapping between D- and $(D+1)$-dimensional models. What we obtain is an identification between the off-shell degrees of freedom of two theories, in D and $(D+1)$ dimensions. Their Lagrangians are usually not related by ordinary dimensional reduction. However, their Q-exact parts are related by the choice of the topological gauge functions. Keeping in mind that the D-dimensional theory is a TQFT that can often be untwisted into a physical supersymmetric theory (e.g. a theory of particles with the physical spin statistics relation identified as the gauge-invariant untwisted fields), we might not be very far from the idea of the holography principle: there is a physical theory with propagating particles on the boundary of the $(D+1)$-dimensional space, in which the theory takes the form of a topological CS-type,one with no particles.

In a second part, we construct classes of theories of higher–dimensional forms for which one can apply the general formalism relating TQFTs in D and $(D+1)$ dimensions: these yet unexplored TQFTs are obtained by combining new types of free self-duality equations with ordinary dimensional reduction. They generalize the simplest examples $dB_k = *dB_k$ for a single field in $D = 2k + 2$ dimensions into $dB_p =^* dC_{D-2-p}$, where both fields B and C are independent, with no restriction on p, except, $p < D - 1$; by dimensional reduction in a lower dimension d, B_p and C_{D-2-p} generate a tower of forms B^i_q and C^i_{d-2-q}, with $q \leq p$. One has, as a starting point, an invariant of the type $\int_D dB_p \wedge dC_{D-2-p}$. Because of [7], we expect that this topological invariant be non zero (it is Z-valued, up to an overall normalization). The free self-duality equations are $dB^i_q =^* dC^i_{d-2-q}$. The forms B^i_q and C^i_{d-2-q} can be arranged into representations of certain Lie algebras, so that i can be seen as a global internal symmetry index. We then observe that a gauging of the global symmetries can often be done, and this allows us to modify the self-duality equations. We eventually get TQFTs involving Yang–Mills fields and forms, which interact by minimal coupling through the "gauged" self-duality equations:

$$DB^i_q =^* DC^i_{d-2-q} \qquad F^a_2 =^* DC^a_{d-3}\,, \tag{1.2}$$

Here D is the covariant derivative with respect to the Yang–Mills field made from the one-forms obtained by the process of dimensional reduction, while F_2 is its curvature. In four dimensions, the last equation is $F_2 =^* F_2$.

In [9], it was shown that the notion of twist of topological symmetries goes beyond the case of the four-dimensional Yang–Mills field theory, for instance in the case of TQFTs with two-forms in six dimensions, and three-forms in eight dimensions, with self-duality equations. It is thus an appealing possibility that the TQFTs with gauged internal supersymmetries exhibited in this paper can be untwisted into new types of theories, with reduced Poincaré supersymmetry. The number of supersymmetry generators might be less than in the standard theories, because of interactions. In an extremal situation, the untwisted supersymmetry has only one generator (corresponding to the BRST Q-operator). On the other hand, the cases of the four- and eight-dimensional Yang–Mills TQFTs show that the full Poincaré supersymmetry can be obtained after untwisting. We list possible internal gauge symmetries for the TQFTs in four and five dimensions, obtained by dimensional reductions of TQFTs in dimensions up to 14. Such a list might be useful for comparison with the list of possible global symmetries of non-trivial interacting field theories in five dimensions [10]. Independently of the possible interpretations of these internal symmetries, the self-duality equations that we have found indicate the existence of yet unexplored TQFTs. They are very natural generalizations of the four-dimensional Yang–Mills TQFT. Finally, the relation of such TQFTs with theories with Poincaré supersymmetry is a very interesting question, which stimulates us to understand the notion of twist in quantum field theory dynamically.

2. Transgression between D- and (D+1)-dimensional TQFTs

The aim of this section is to indicate an algorithm for defining a quantum field theory related to first-order actions in $(D+1)$ dimensions. One starts from an "ordinary" TQFT in D dimensions. It is defined from the knowledge of a BRST operator Q acting on a set of fields, for instance p-form gauge fields, their ghosts and topological ghosts. Q and the Q-invariant action I_D satisfy

$$Q^2 = 0 \text{ modulo gauge transformations of forms in } D \text{ dimensions} \tag{2.1}$$

$$I_D = I_{D\,\mathrm{top}}[\varphi] + \{Q, \ldots\}\,, \tag{2.2}$$

where $I_{D\,\mathrm{top}}$ is a classical topological invariant, which is a function of only the classical fields φ, so the classical Lagrangian is locally a d-exact D-form. The set of fields on which Q acts can be found from methods as in [6][9], and the terms denoted as $\ldots$ in the Q-exact term are topological gauge functions as those defined, e.g in [11].

Our observation is that the fields of the D-dimensional TQFT can be redefined as elements of a multiplet in $(D+1)$-dimensional space, with a modified definition of Q: Q acts on the classical fields of the $(D+1)$-dimensional theory as in (1.1) and it satisfies on all fields the following relation:

$$Q^2 = \partial_{D+1} \text{ modulo gauge transformations of forms in } (D+1) \text{ dimensions.} \tag{2.3}$$

Moreover, one has a Q-invariant action

$$I_{D+1} = I_{D+1\,\mathrm{CS}}[\varphi, \Psi] + \{Q, \ldots\}\,. \tag{2.4}$$

The role of the Q-exact term in the right-hand side of (2.4) is to define the theory at the quantum level: the $\ldots$ terms are in correspondence with the topological gauge functions of the TQFT in D dimensions, and they eventually provide a Lorentz-invariant contribution to the action in $(D+1)$ dimensions.[2] with a dependence on the field derivatives which is second order for the bosons and first order for the fermions. The other term in (2.4), $I_{D+1\mathrm{CS}}[\varphi,\Psi]$, is the sum of a classical first-order action and of a ghost-dependent action. This part of the Lagrangian is Q-invariant, but it is neither Q-exact nor d-exact as the D-dimensional theory Lagrangian. The expression $I_{D+1\mathrm{CS}}[\varphi,0]$ can be called the classical bosonic CS action. Without the addition of the Q-exact term to $I_{D+1\mathrm{CS}}[\varphi,\Psi]$, the definition of the theory is difficult, and may be ambiguous (since the Hamiltonian of $I_{D+1\mathrm{CS}}[\varphi,0]$ identically vanishes). By a suitable choice of the ... terms in (2.4), the Q-exact term determines a meaningful local quantum theory, because it adds to $I_{D+1\mathrm{CS}}[\varphi,\Psi]$ a sum of squared curvatures. One can adopt the point of view that $I_{D+1\mathrm{CS}}$ can be factored out in the measure of the path integral, as for instance the θ term in QCD.

One generally finds observables other than $I_{D+1\mathrm{CS}}[\varphi,\Psi]$ in the cohomology of Q in $(D+1)$ dimensions . They must be searched for case by case, by lifting to a $(D+1)$-dimensional space the p-forms on which Q acts in D dimensions.

In what follows, for the sake of notational simplicity, we examine the case of two-form gauge fields, and see how a CS-type theory for two-forms in five dimensions is related to a TQFT in four dimensions depending on two-forms *and* scalars. Then, we generalize the construction.

Our formula holds for fields with no interactions. They can be adapted to the cases of systems whose interactions are described by non-free curvatures satisfying Bianchi identities.

2.1. The example of two-form gauge fields

As shown in [9], the field spectrum of a TQFT for a two-form gauge field B_{ij}, where $i,j,\dots$ are D-dimensional indices is described by the following TQFT multiplet

$$(B_{ij},\Psi^1_{[ij]},\chi^{-1}_\alpha,H_\alpha,\eta^{-1}_i,\Phi^3,\Phi^{-3},X^1,\Phi^2_i,\Phi^{-2}_i,\eta^2,\eta^{-2})\,. \tag{2.5}$$

The definition of the topological BRST operator Q is

$$\begin{aligned}
QB_{ij} &= \Psi^1_{[ij]} & Q\Psi^1_{[ij]} &= \partial_{[i}\Phi^2_{j]}\\
Q\Phi^2_i &= \partial_i\Phi^3 & Q\Phi^3 &= 0\\
Q\chi^{-1}_\alpha &= H_\alpha & QH_\alpha &= 0\\
Q\Phi^{-2}_i &= \eta^{-1}_i & Q\eta^{-1}_i &= 0\\
Q\Phi^{-3} &= \eta^{-2} & Q\eta^{-2} &= 0\\
QX^1 &= \eta^2 & Q\eta^2 &= 0\,.
\end{aligned} \tag{2.6}$$

The upper index denotes the ghost number. Its value modulo 2 determines the statistics of fields. The index α of the antighost χ^{-1}_α runs over the number of independent

[2] This holds true despite the fact that the expression of Q violates Lorentz invariance in $(D+1)$ dimensions, as shown by (2.3).

topological gauge functions (e.g. gauge-invariant generalized self-duality conditions as in [11]) that the TQFT imposes to define a local dynamics for the two-form gauge field. The TQFT action has the form:[3]

$$I_D = I_{\text{top}}[B] + \int_D \{Q, \chi_\alpha^{-1}(F_\alpha(B) + \frac{1}{2}H_\alpha) + \Phi_i^{-2}\partial_j \Psi^1_{[ij]} + \Phi^{-3}\partial_i \Phi_i^2 + X^1 \partial_i \Phi_i^{-2}\}\,. \quad (2.7)$$

The BRST algebra (2.6) satisfies (2.1).

The action (2.7) has been studied for $D = 6$ in [9]. In this case, self-duality equations can be derived for a pair of two-forms and thus the gauge functions can be determined. After the elimination of fields with algebraic equations of motion, one has the interesting result that the TQFT symmetry can be related by a twist to Poincaré supersymmetry [9].

The definition of a theory for two-forms in $(D+1)$ dimensions follows from the possibility of rearranging by linear combinations all fields of the D-dimensional Lorentz covariant TQFT multiplet (2.5) into a $(D+1)$-dimensional Lorentz-covariant multiplet

$$(B_{\mu\nu}, \Psi^-_{[\mu\nu]}, \phi^+_\mu, \phi^-, \Phi^-, X^-, \eta^+, \chi^-_\alpha, H_\alpha)\,. \quad (2.8)$$

Let us detail this multiplet. The $(D+1)$-dimensional Greek indices can be decomposed as $, i$. The mapping between the fields in (2.5) and (2.8) is as follows:

$$\begin{aligned} B_{ij}, \Phi_i^2, \Phi_i^{-2}, \eta^2 &\to B_{\mu\nu}, \phi^+_\mu \\ \Psi^1_{[ij]}, \eta_i^{-1} &\to \Psi^-_{[\mu\nu]} \\ \Phi^3, \Phi^{-3}, X^{-1}, \eta^{-2}, \chi^-_\alpha &\to \phi^-, \Phi^-, X^-, \eta^+, \chi^-_\alpha\,. \end{aligned} \quad (2.9)$$

This mapping violates modulo 2 the ghost-number conservation of the D-dimensional theory (this explains our unified notation by an upper index $\pm$ for the positive and negative statistics of fields in (2.8)). The BRST operator Q, which satisfies (2.3), is

$$\begin{aligned} QB_{\mu\nu} &= \Psi^-_{[\mu\nu]} & Q\Psi^-_{[\mu\nu]} &= \partial_{[}B_{\mu\nu]} + \partial_{[\mu}\phi^+_{\nu]} \\ Q\phi^+_\mu &= \Psi^-_{[]} + \partial_\mu\phi^- & Q\phi^- &= \phi^+ \end{aligned} \quad (2.10)$$

and

$$\begin{aligned} Q\chi^-_\alpha &= H_\alpha & QH_\alpha &= \partial\chi^-_\alpha \\ Q\Phi^- &= \eta^+ & Q\eta^+ &= \partial\Phi^- \\ QX^- &= \eta^+ & Q\eta^+ &= \partial X^-\,. \end{aligned} \quad (2.11)$$

In comparison, in the Yang–Mills case, the fields of the D-dimensional Yang–Mills TQFT $(A_i, \Psi_i^1, \Phi^2, \Phi^{-2}, \eta^{-1}, \chi^{-1})$ can be rearranged into the $(D+1)$-dimensional multiplet $(A_\mu, \Psi^-_\mu, \phi^+, \chi^{-1})$ and the equations analogous to (2.10) and (2.11) are [8]:

$$\begin{aligned} &QA_\mu = \Psi^-_\mu \;\; Q\Psi^-_\mu = F + \partial_\mu\phi^+ \\ &Q\phi^+ = \Psi^- \\ &Q\chi^- = H \quad QH = \partial\chi\,. \end{aligned} \quad (2.12)$$

[3]We assume the existence of gauge functions $F_\alpha(B)$ without specifying their expressions.

Next comes the question of determining a Q-invariant action in $(D+1)$ dimensions. Equation (2.4) indicates two distinguishable terms. The Q-exact term exists in $(D+1)$ dimensions if there are generalized self-duality equations in D dimensions; the Q-invariant and not Q-exact term exists if there is a Chern–Simons-like term in $(D+1)$ dimensions. Choosing $D=4$, one finds for instance that a minimal set of fields involving two-forms consists of a pair of two two-forms B_{ij} and $\bar{B}_{ij}$ and two zero-forms S and $\bar{S}$. This is justified by the existence of a TQFT for a three-form gauge field in eight dimensions with ordinary self-duality gauge conditions [6][9] and by its dimensional reduction according to

$$\begin{aligned} D=8 &: B_3 \\ D=7 &: B_3\ B_2 \\ D=6 &: B_3\ 2B_2\ B_1 \\ D=5 &: B_3\ 3B_2\ 3B_1\ S \\ D=4 &: B_3\ 4B_2\ 6B_1\ 4S \end{aligned} \tag{2.13}$$

(The generality of this reduction will be addressed in the last section). The set of fields for $D=4$ includes: a three-form gauge field, which determines a TQFT by itself (it carries no degree of freedom and can be used to allow spontaneous symmetry breaking in scale-invariant theories of three-branes [12]); six one-forms, four two-forms and four zero-forms. The dimensionally reduced self-duality equations are

$$F^a_{ij} = \epsilon_{ijkl} F^{akl} \tag{2.14}$$

$$F^b_{ijk} = \epsilon_{ijkl} F^b_l\,, \tag{2.15}$$

where i,j,k,l are now four-dimensional indices. The internal indices a and b run over six and four possibilities respectively. The two-forms F^a_2 are the curvature of the one-forms B^a_1; F^a_3 and F^a_1 are the curvature of the two-forms B^a_2 and zero-forms S^a respectively. The geometric nature of the indices a,b is an interesting question. They can be identified as $SO(4)$ indices: one can assemble the six one-forms B^a_1 as components of a Yang–Mills field valued in the Lie algebra of $SO(4)$ and B^b_2 and S^b as the components of a vector of $SO(4)$. Thus, one can define F^a_{ij} as the non-Abelian curvature of A, and F_1 and F_3 as the covariant derivatives of the zero-forms S^a and two-forms B^a_2. Having a topological theory with $SO(4)$ Yang–Mills invariance suggests a link to four dimensional topological gravity.

We are now ready to construct a five-dimensional theory associated to that of a four dimensional theory for B_{ij}, $\bar{B}_{ij}$, S and $\bar{S}$, as an illustration of the algorithm relating D and $(D+1)$ theories.

Because we have introduced the scalars S and $\bar{S}$, we must complement (2.6) for $D=4$ by

$$QS = \Psi_S \qquad\qquad Q\Psi_S = 0 \tag{2.16}$$

and (2.10) for $D=5$ by

$$QS = \Psi_S \qquad\qquad Q\Psi_S = \partial_5 S\,, \tag{2.17}$$

with analogous equations for $\bar{S}$.

The four-dimensional theory relies on the "mixed" self-duality equations:

$$\partial_i S = \epsilon_{ijkl}\partial_j \bar{B}_{kl} \tag{2.18}$$

$$\partial_i \bar{S} = \epsilon_{ijkl}\partial_j B_{kl} \tag{2.19}$$

The classical topological action is

$$I_{4\ \mathrm{top}} = \int_4 dS \wedge d\bar{B}_2 + d\bar{S} \wedge dB_2\,. \tag{2.20}$$

Using the gauge functions (2.18) and (2.19) in the standard way (there are, actually, four antighosts κ_a^{-1} and four Lagrange multipliers H_a for the system (S, B_2), which correspond to the 1 degree of freedom for S plus the $3 = 6-4+1$ off-shell gauge-invariant degrees of freedom for the two-form B_2 in four dimensions), one gets a four-dimensional TQFT action

$$I_4 = \int_4 d^4x(|\partial_i S|^2 + |\partial_i \bar{S}|^2 + |\partial_{[i}\bar{B}_{jk]}|^2 + |\partial_{[i}B_{jk]}|^2 + \ldots)\,. \tag{2.21}$$

The terms made explicit in this action are obtained by squaring the gauge functions. The coefficients are chosen such that the mixed terms obtained in this operation cancel against $I_{4\ \mathrm{top}}$, leaving the squared curvatures. The terms $\ldots$ stand for the various ghost-depending terms analogous to those made explicit in [9] for the six-dimensional TQFT of two-forms. We expect that the more complete TQFT action using all fields appearing in (2.13) for $D = 4$ can be untwisted in an ordinary supersymmetric action, since this is presumably the case for the eight-dimensional action for a three-form [9].

The five-dimensional action (2.4) has a CS term

$$I_{5\ \mathrm{CS}} = \int_5 B_2 \wedge d\bar{B}_2 + d^5x\epsilon^{ijkl5}(\bar{\varphi}_i^+\partial_{[j}B_{jk]} + \varphi_i^+\partial_{[j}\bar{B}_{jk]} + \bar{\Psi}_{ij}\Psi_{kl})\,. \tag{2.22}$$

It depends on the four-dimensional two-forms lifted to five dimensions, but not on the scalars S. The terms depending on φ and $\bar{\varphi}$ are related to Gauss's law for the CS-like term $B_2 \wedge d\bar{B}_2$.

The expression of the Q-exact term in (2.4) is inspired from the four-dimensional gauge functions (2.18) and (2.19). It is

$$\begin{aligned}\int d^5x\ \{\ Q, \chi_i(\epsilon^{ijkl5}\partial_j\bar{B}_{kl} + \partial_i\bar{S} + \frac{1}{2}H_i) + \bar{\chi}_i(\epsilon^{ijkl5}\partial_j B_{kl} + \partial_i S + \frac{1}{2}\bar{H}_i)\\ +\Psi^-_{\mu\nu}(\partial_{[5}\bar{B}_{\mu\nu]} + \partial_{[\mu}\bar{\varphi}^+_{\nu]} + \Psi^-_S\partial_5\bar{S}) + \bar{\Psi}^-_{\mu\nu}(\partial_{[5}B_{\mu\nu]} + \partial_{[\mu}r\varphi^+_{\nu]} + \bar{\Psi}^-_S\partial_5 S)\ \}\,.\end{aligned} \tag{2.23}$$

Adding the two terms (2.22) and (2.23), we obtain

$$\begin{aligned}I_5 = \int_5 B_2 \wedge d\bar{B}_2 + d^5x\epsilon^{ijkl5}(\bar{\varphi}_i^+\partial_{[j}B_{kl]} + \varphi_i^+\partial_{[j}\bar{B}_{kl]} + \bar{\Psi}_{ij}\Psi_{kl})\\ +d^5x(|\partial_\mu S|^2 + |\partial_\mu\bar{S}|^2 + |\partial_{[\mu}\bar{B}_{\nu\rho]}|^2 + |\partial_{[\mu}B_{\nu\rho]}|^2 + \ldots)\,.\end{aligned} \tag{2.24}$$

The $\ldots$ terms stand for terms depending on the topological ghosts, which are lengthy to write. Their presence enforces the topological behaviour of the theory because there are compensations between bosons and fermions. What truly counts when one computes Q-invariant observables are the contributions of the zero modes that encode the topological information of the theory. In the absence of the Q-exact term (2.23) it seems difficult to directly quantize the action (2.22) alone, and the compensation between bosons and fermions is not explicit. In this sense, the action (2.23) is a useful topogical gauge-fixing term for the theory.

We could have considered a gauge in which the Q-exact terms are such that, after using the constraints coming from field equations, only the term $\int_5 B_2 \wedge d\bar{B}_2$ remains, while all fermions are spectators. We believe that the definition of the system from this sole term could be ambiguous while the complete action (2.24) determines a predictive QFT.

2.2. Generalization

We generalize to the case of a p-form B_p with curvature F_{p+1}. We denote as $X_{,r}$ the $X_{,\mu_1\ldots\mu_r}$ component of a $(r+1)$-form X_{r+1}. Then, the following definition generalizes the expression (2.10) of Q that we have established for the case of a two-form

$$\begin{aligned} QB_p &= \Psi_p^- \\ Q\Psi_p^- &= F_{,p} + d\phi_{p-1}^+ \\ Q\phi_{p-1}^+ &= \Psi_{,p-1}^- + d\phi_{p-2}^- \\ Q\phi_{p-2}^- &= \phi_{,p-2}^+ + d\phi_{p-3}^- \\ &\ldots \\ Q\phi_O^\pm &= \phi^\mp \,. \end{aligned} \tag{2.25}$$

We could as well consider a collection of p-forms, with different values of p. By construction, the operator Q defined in (2.25) satisfies the relation (2.3). One can then obtain the Q-exact term of the $(D+1)$-dimensional theory if one has self duality equations for the p-form in D dimensions. One must also investigate whether one has Q-invariant and not Q-exact terms depending on the p-forms to eventually get a (supersymmetric) Chern–Simons action generalizing (2.22).

As new examples, we can start from pairs of independent forms B_p and C_{D-2-p}, with different values of p. Indeed, one may consider combinations of the following actions in D and $(D+1)$ dimension

$$I_D = \int_D dB_p \wedge dC_{D-2-p} + \{Q, \ \} \,, \tag{2.26}$$

with their possible $(D+1)$ counterparts

$$I_{D+1} = \int_{D+1} I_{D+1\mathrm{CS}}[B, C, \Psi] + \{Q, \ \} \,. \tag{2.27}$$

The expressions for Q satisfy (2.1) in (2.26) and (2.3) in (2.27). The expression of $I_{D+1\mathrm{CS}}[B, C, \Psi]$ depends on the particular cases and, to our knowledge, cannot be inferred from the existence of invariants in D dimensions.

There is another $(D-1)$-dimensional TQFT action, which is :

$$I_{D-1} = \int_{D-1} B_p \wedge dC_{D-2-p} + \{Q, \ \} . \tag{2.28}$$

Its study would imply the analysis of the TQFTs in $D-2$ dimensions.

The action (2.26) induces a TQFT that parallels the four-dimensional Yang–Mills case if one uses uses as a gauge function the duality condition between the curvatures F_{p+1} and G_{d-1-p} of the p-form gauge field B_p and B_{d-2-p}:

$$F_{p+1} =^* G_{d-1-p} . \tag{2.29}$$

The four-dimensional Yang–Mills case is exceptional, since the self-duality equation only involves one field (see [11] for an attempt at classifying the possible TQFT with a single field).

Having at our disposal theories defined by (2.26) is a new feature. They depend on two independent fields which can be related by a self-duality relation between their curvatures. One can write all sorts of descent equations, which formally determine observables and it would be interesting to investigate in which cases these TQFTs can be untwisted into supersymmetric theories.

Up to renamings, the field degrees of freedom are the same for the actions (2.26) and (2.27). Let us conclude this section by the following remark. The ordinary ghosts and topological ghosts of B_p and C_{D-2-p} can be unified in the expansion in ghost number of generalized forms $\tilde{B}_p$ and $\tilde{C}_{D-2-p}$ and $\tilde{F}_{p+1}$ and $\tilde{G}_{D-1-p}$. For these forms, the grading is the sum of D-dimensional Lorentz degree and ghost number. If, as explained in [13], we accept fields with negative ghost number in the generalized forms, one finds that the components with negative ghost number in $\tilde{G}_{D-1-p}$ are the Batalin–Vilkovisky antifields of the fields with positive ghost number in $\tilde{B}_p$, while the components with negative ghost number in $\tilde{B}_p$ are the Batalin–Vilkovisky antifields of the fields with positive ghost number in $\tilde{G}_{D-1-p}$. There are analogous relations between the field contents of $\tilde{C}_{D-2-p}$ and $\tilde{F}_{p+1}$.

3. Applications and gauging of free self-duality equations by dimensional reduction

As an example for the action (2.26), we have in $D = 11$ the following classical "topological" term, inspired by the $N = 1, D = 11$ supergravity:

$$I_{11} = \int_{11} dC_3 \wedge dB_6 + \{Q, \ \} \tag{3.1}$$

and the gauge function in eleven dimensions

$$dC_3 =^* dB_6\,. \tag{3.2}$$

Interactions can be introduced, e.g. by improving the gauge functions by addition of a gravitational or Yang–Mills Chern class of rank four [6]. The possibility of this modification relies on the fact that C_3 can be defined as a connection rather than as a form, as explained in [7]. Thus we expect a TQFT in $D = 11$ and $D = 12$, depending on a three-form and a six-form B_6, by the method of section 2.

Let us now consider the generic case for (2.26), with the free self-duality equation

$$dB_p =^* dC_{D-2-p} \tag{3.3}$$

and see in more detail the possibility of dimensional reduction briefly mentioned in section 2.1 for the TQFTs determined from $\int_D dB_p \wedge dC_{D-2-p}$. The foregoing analysis includes the particular case $D = 2p+2$, in which a single one-form $B = C$ is sufficient to write the self-duality equation.

Dimensional reduction increases the number of forms of a given degree according to the property that a form of degree p in d dimensions gives in $(d-1)$ dimensions a form of degree p and a form of degree $p-1$. The coefficients in the following Pascal triangle describe the number of forms B_p, B_{p-1}, etc, obtained by dimensional reduction of a p-form B_p from D to any given lower dimension $d = D - \Delta D$:[4]

$$\begin{array}{cccccccccccccc}
 & B_p & B_{p-1} & B_{p-2} & B_{p-3} & B_{p-4} & B_{p-6} & B_{p-7} & \ldots & & & & & \\
D & 1 & & & & & & & & & & & & \\
D-1 & 1 & 1 & & & & & & & & & & & \\
D-2 & 1 & 2 & 1 & & & & & & & & & & \\
D-3 & 1 & 3 & 3 & 1 & & & & & & & & & \\
D-4 & 1 & 4 & 6 & 4 & 1 & & & & & & & & \\
D-5 & 1 & 5 & 10 & 10 & 5 & 1 & & & & & & & \\
D-6 & 1 & 6 & 15 & 20 & 15 & 6 & 1 & & & & & & \\
D-7 & 1 & 7 & 21 & 35 & 35 & 21 & 7 & 1 & & & & & \\
D-8 & 1 & 8 & 28 & 56 & 70 & 56 & 28 & 8 & 1 & & & & \\
D-9 & 1 & 9 & 36 & 84 & 126 & 126 & 84 & 36 & 9 & 1 & & & \\
D-10 & 1 & 10 & 45 & 120 & 210 & 252 & 210 & 120 & 45 & 10 & 1 & & \\
D-11 & 1 & 11 & 55 & 165 & 330 & 462 & 462 & 330 & 165 & 55 & 11 & 1 & \\
D-12 & 1 & 12 & 66 & 220 & 495 & 792 & 924 & 792 & 495 & 220 & 66 & 12 & 1 \\
\ldots & & & & & & & & & & & & &
\end{array} \tag{3.4}$$

Clearly, the general formula for the number of $(p-\Delta p)$-forms obtained by dimensional reduction of a p-form from D to $d = D - \Delta D$ dimensions is $C^{\Delta D}_{\Delta p}$.

Table (3.4) indicates that, by dimensionally reducing from D to d a pair of independent forms B_p and C_{D-2-p}, one gets $2C^{\Delta D}_{\Delta p}$ forms of degree $p - \Delta p$ and $2C^{\Delta D}_{\Delta p}$ forms of

[4] The entries in the table vanish for forms that do not exist in the chosen reduced dimension.

degree $d-2-(p-\Delta p)$. For $D \neq 2p+2$, B_p and C_{D-2-p} contribute in an asymmetrical way to the total number of forms of a given degree in dimension d. The matching between the number of q-forms and $(d-2-q)$-forms generated by dimensional reduction of B_p and C_{D-2-p} is important: it leads to multiple self- duality equations in d dimensions

$$dB^i_q =^* dC^i_{d-2-q}\,, \tag{3.5}$$

where the index i runs over the number of possibilities indicated by the combinatorial factor $C^{\Delta D}_{\Delta p}$.

We will use the fact that the d-dimensional forms B^i_q and C^i_q can be arranged in a $SO(d)$-covariant way as elements of representations of a Lie algebra G. There are generally several possibilities for G. If the number of one-forms B^a_1 and C^a_1 equals the dimension of the adjoint representation of G, we can identify B^a_1 and C^a_1 as the components of a G-valued Yang–Mills field A_1, and gauge the global symmetry. This supposes that *all* other forms B^i_q and C^i_{d-2-q} fit in various representations of G. If it is the case, denoting by f^a_{bc} the structure coefficients of G and by T^i_{ja} the matrix elements of a given representation X^i of G, the gauging simply amounts to changing the free dimensionally reduced self-duality equations $dB^i_q =^* dC^i_{d-q-2}$ into the non-Abelian equations

$$\begin{aligned} &dB^i_q + T^i_{ja}A^a_1 B^j_q =^* (dC^i_{d-q-2} + T^i_{ja}A^a_1 C^j_{d-q-2}) \\ &dA^a_1 + \frac{1}{2} f^a_{bc}A^b_1 A^c_1 =^* (dC^a_{d-3} + f^a_{bc}A^b_1 C^c_{d-3})\,. \end{aligned} \tag{3.6}$$

These equations are the self-duality equations (1.2) quoted in the introduction.

Imposing the non-Abelian self-duality equations (3.6) as topological gauge conditions of a TQFT with a Q-invariant action is straightforward from the point of view of BRST quantization. The result is an action

$$\int d^dx \ \mathrm{tr}(|F^a|^2 + \sum_q |DB^i_{d-q-2}|^2 + |DB^i_q|^2 + \text{supersymmetric terms})\,. \tag{3.7}$$

The gauge function involving the Yang–Mills curvature $F = dA + AA$ in (3.6) is a generalization of the three-dimensional Bogomolny equation between a Yang–Mills field and a Lie algebra valued scalar field. When the dimensional reduction is done in four dimensions, the gauge function on F reduces to the self-duality condition $F =^* F$.

In the process that we have just described, the interactions only arise through minimal coupling to a Yang–Mills field. However, if we were to modify the free self-duality equations before dimensional reduction, for instance by adding to the free curvatures Chern–Simons-like terms made of external products of various fields, the gauging after dimensional reduction would still be consistent. One example is to begin with a ten-dimensional self-duality equation for a four-form B_4 interacting with a pair of real two-forms B_2 and B'_2, $G_5 =^* G_5$, with $G_5 = dB_4 + dB_2 \wedge B'_2 - dB'_2 \wedge B_2$. The curvatures of both two-forms are purely Abelian, $G_3 = dB_2$ and $G'_3 = dB'_2$. By dimensional reduction in four dimensions we obtain a TQFT whose gauge functions are self-duality equations of the type (2.18) and (2.19). The interacting terms can be worked out from the definition of G_5. Another example is to begin with self-duality equations involving p-forms with Chapline–Manton-type curvatures $G_{p+1} = dB_p + Q_{p+1}(A, F)$, where $Q_{p+1}(A, F)$ is a Yang–Mills Chern–Simons term. We expect a dimensionally reduced theory with a gauge symmetry equal to the product of the original Yang–Mills symmetry by the symmetries coming from dimensional reduction.

Note that a direct construction can yield more invariants in d dimensions than those provided by dimensional reduction of the invariants in D dimensions. For instance, if we start in $D = 8$ with a three-form B_3, dimensional reduction gives four one-forms B_1^a in $d = 4$; the invariant $\int_8 dB_3 \wedge dB_3$ gives $\int_4 \epsilon_{abcd} F_2^{ab} \wedge F_2^{cd}$, where the internal indices $a, b, ...$ are $SO(4)$ indices. However, the invariant $\int_4 F_2^{ab} \wedge F_{2ab}$ is not obtained by dimensional reduction, and must be introduced directly in the four-dimensional theory.

Let us also note that the global symmetry $SO(\Delta D)$, which generically occurs in dimensional reduction, is not the one that is generally used for the gauging. There is however, the exceptional case of the dimensional reduction of a three-form, which gives $\Delta D(\Delta D - 1)/2$ one-forms, which matches the dimension of the adjoint representation of $SO(\Delta D)$.

As examples, let us first examine the five- and four-dimensionsal gauge symmetries that are obtained by dimensional reductions of k-forms with a self-duality equation $dB_k = *dB_k$ in $D = 2k + 2$ dimensions. Using (3.4), we have, for $d = 5$:

$$\begin{aligned}
D = 6 &: B_2 \to d = 5 : B_3,\ B_2 \\
D = 8 &: B_3 \to d = 5 : B_3,\ 3B_2,\ \mathbf{3B_1},\ S \\
D = 10 &: B_4 \to d = 5 : B_4,\ 5B_3,\ 10B_2,\ \mathbf{10B_1},\ 5S \\
D = 12 &: B_5 \to d = 5 : B_5,\ 7B_4,\ 21B_3,\ 35B_2,\ \mathbf{35B_1},\ 21S \\
D = 14 &: B_6 \to d = 5 : 9B_5,\ 36B_4,\ 84B_3,\ 126B_2,\ \mathbf{126B_1},\ 84S
\end{aligned} \tag{3.8}$$

and for $d = 4$:

$$\begin{aligned}
D = 6 &: B_2 \to d = 4 : B_2,,\ \mathbf{2B_1},\ S \\
D = 8 &: B_3 \to d = 4 : B_3,\ 4B_2,\ \mathbf{6B_1},\ 4S \\
D = 10 &: B_4 \to d = 4 : B_4,\ 6B_3,\ 15B_2,\ \mathbf{20B_1},\ 15S \\
D = 12 &: B_5 \to d = 4 : 8B_4,\ 28B_3,\ 56B_2,\ \mathbf{70B_1},\ 56S \\
D = 14 &: B_6 \to d = 4 : 45B_4,\ 120B_3,\ 210B_2,\ \mathbf{252B_1},\ 210S\,.
\end{aligned} \tag{3.9}$$

We have put in bold characters the number of one-forms that occur for each decomposition. Thus, possible internal symmetries such that the number of the one- forms fits the number of generators of their Lie algebra are, in $d = 5$:

$$\begin{aligned}
3B_1 &: SU(2) \\
10B_1 &: SO(5);\ SO(4) \times SU(2) \times U(1) \\
35B_1 &: SU(6);\ SO(7) \times G_2 \\
126B_1 &: E_6 \times SU(7);\ SU(8) \times SU(8)
\end{aligned} \tag{3.10}$$

and in $d=4$:

$$\begin{aligned}
6B_1 &: SO(4)\\
20B_1 &: SO(5)\times SO(5);\ SO(4)\times G_2\\
70B_1 &: SU(6)\times SU(6)\ ;SU(2)\times U(1)\times SO(12);\ ;\ SO(8)\times SO(8)\times G_2\\
252B_1 &: E_8\times SU(2)\times U(1);\ (E_6\times SU(7))^2\,.
\end{aligned} \tag{3.11}$$

One can verify in each case that the other fields can be arranged into representations of the Lie algebra G as "matter" fields. Let us work out a few examples. For $d=5$ with $G=SO(5)$, the 10 one-forms coming from the dimensional reduction of a four-form in ten dimensions are identified as a Yang–Mills field for $SO(5)$ while, the two-form are also valued in the Lie algebra of $SO(5)$; the 5 three-forms and zero-forms B_3 and S are in the fundamental representations and B_4 is a singlet. For $d=5$ and $G=SO(7)\times G_2$, the $35=21+14$ one-forms coming from the dimensional reduction of a five-form in 12 dimensions build a Yang–Mills field valued in the adjoint representations of $G=SO(7)\times G_2$; the 35 two-forms can be arranged in the same representation; the 21 "matter" three-forms and zero-forms B_3 and S can be identified either as $\underline{21}$, $\underline{7}\oplus\underline{7}\oplus\underline{7}$ or $\underline{7}\oplus\underline{14}$ of $SO(7)\times G_2$; the seven four-forms B_4 are in the fundamental representation of $SO(7)$ or G_2 and B_5 is a singlet. This five-dimensional case is of interest, since it contains a five-form giving an action $\int_5 B_5$, which can be used as a Wess–Zumino action to compensate for a possible five-dimensional anomaly, and seven four-forms B_4, which can be used to give a scale [12].

Let us consider theories with pairs of fields, adapted to a starting point in twelve dimensions, which is tantalizing from the point of view of F theory. For instance, if we start from a six-form and a three-form in twelve dimensions, in the context of the CS supersymmetric theory associated to an eleven-dimensional TQFT stemming from (3.1). Table (3.4) gives the following field spectrum in four dimensions

$$\begin{aligned}
D=12:\ & B_6\to d=4\ :\ 28B_4,\ 56B_3,\ 70B_2,\ \mathbf{56B_1},\ 28S\\
D=12:\ & B_3\to d=4\ :\ 1B_3,\ 28B_2,\ \mathbf{28B_1},\ 56S\,.
\end{aligned} \tag{3.12}$$

We see that one has the possibility of having a four-dimensional theory with the one-forms in the adjoint representations $\underline{28}\oplus\underline{28}\oplus\underline{28}$ of $SO(8)\times SO(8)\times SO(8)$.

Another possibility is to start from a TQFT with a six-form and a four-form in twelve dimensions, with the action $\int_{12} dB_6\wedge dB_4$. Then

$$\begin{aligned}
D=12:\ & B_6\to d=4\ :\ 28B_4,\ 56B_3,\ 70B_2,\ \mathbf{56B_1},\ 28S\\
D=12:\ & B_4\to d=4\ :\ 1B_4,\ 28B_3,\ 28B_2,\ \mathbf{56B_1},\ 70S\,.
\end{aligned} \tag{3.13}$$

In this case, the gauge symmetry can be chosen for example as $[SO(8)]^4$ for the four-dimensional theory.

Finally, we can consider a theory with a seven-form and a three-form TQFT in twelve dimensions, with the action $\int_{12}(dB_7\wedge dB_3+dB_3\wedge dB_3\wedge dB_3)$. Then,

$$\begin{aligned}
D=12:\ & B_7\to d=4\ :\ 56B_4,\ 70B_3,\ 56B_2,\mathbf{28B_1},\ 8S\\
D=12:\ & B_3\to d=4\ :\ 1B_3,\ 28B_2,\ 28B_1,\ \mathbf{28B_1},\ 56S\,.
\end{aligned} \tag{3.14}$$

The gauge symmetry can be chosen for example as $SO(8)\times SO(8)$ or $SO(3)^6\times SO(10)^2$ for the four-dimensional theory.

We conclude this section by the following remarks. The gauging of global internal symmetries in TQFTs when we dimensionally reduce the free self-duality equation shown above is analogous to that occurring in supergravities. For supergravities, string inspired arguments justify this gauging [14].

Untwisting the TQFTs obtained by dimensional reduction of free theories as those given by (3.8) and (3.9) can give theories with ordinary Poincaré supersymmetry, by redefining the topological ghosts as the elements of classified multiplets of supersymmetry. There are examples of this in [9]. In this case, the free Q-invariant actions that use the Abelian self-duality equations as topological gauge functions satisfy Poincaré supersymmetry after untwisting the fields. The generator of the Q-symmetry appears as a particular combination of the generators of supersymmetry, i.e. the parameter of the BRST symmetry is a one-dimensional projection of the spinorial parameter of Poincaré supersymmetry. The Q-symmetry survives, by construction, the untwisting procedure and the introduction of interactions. However, when one enforces in a Q-invariant way the gauging of internal symmetries by modification of the gauge functions, it is unclear to us if the the *full* supersymmetry of the action is still existing after untwisting the fields as in the free case. If a reduction of supersymmetry occurs, it could parallel the analogous property at work when one tries to find an effective supersymmetric theory in the world volumes of branes. In our context, the extreme case would be that only the Q symmetry is present in the interactive theories.

Acknowledgments

We wish to thank S. Ferrara, C. Kounnas and S. Shatashvili for discussions.

References

[1] A partial list of recent works is:
E. Witten, *Some Comments on String Dynamics, Contribution to STRINGS95: Future Prospects in String Theory, Los Angeles, CA, 13-18 March 1995*, hep-th/9507121 ;
O. Ganor and A. Hanany, *Small E(8) Instantons and Tensionless Noncritical strings*, *Nucl. Phys.* **B 474** (1996) 122 [hep-th/9602120];
N. Seiberg and E. Witten, *Comments on String Dynamics in Six Dimensions*, *Nucl. Phys.* **B 471** (1996) 121 [hep-th/9603003];
N. Seiberg, *Five Dimensional SUSY Field Theories, Non-trivial Fixed Points and String Dynamics*, *Phys. Lett.* **B 388** (1996) 753 [hep-th/9608111];
N. Seiberg, *Non-trivial Fixed Points of The Renormalization Group in Six Dimensions*, *Phys. Lett.* **B 390** (1997) 169 [hep-th/9609161];
M. Bershadsky and C. Vafa, *Global Anomalies and Geometric Engineering of Critical Theories in Six Dimensions*, hep-th/9703167;

J.L. Blum and K. Intriligator, *New Phases of String Theory and 6d RG Fixed Points via Branes at Orbifold Singularities*, *Nucl. Phys.* **B 506** (1997) 199 [hep-th/9705044];
N. Seiberg, *Notes on Theories with 16 Supercharges*, hep-th/9705117, *An updated version of lectures presented at the Jerusalem Winter School on Strings and Duality.* To appear in the Proceedings of the Trieste Spring School (March 7–12, 1997).

[2] G. 't Hooft, *Dimensional Reduction in Quantum Gravity* in *Salamfest* 1993, p.284, gr-qc/9310026;
L. Susskind, *The world as a Hologram* , *J. Math. Phys.* **36** (1995) 6377 [hep-th/9409089].

[3] P. Horava, *M-Theory as a Holographic Field Theory*, hep-th/9712130.

[4] E. Witten, *Topological Quantum Field Theory*, *Comm. Math. Phys.* **117** (1988) 353.

[5] L. Baulieu and I. M. Singer, *Topological Yang–Mills Symmetry*, LPTHE 88-18, in *Annecy Field Theory '88*, p. 12, Annecy, France.;
L. Baulieu, *On the Symmetries of Topological Quantum Field Theories*, *Int. J. Mod. Phys.* **A 10** (1995) 4483 [hep-th/9504015];
R. Dijkgraaf and G. Moore, *Balanced Topological Field Theories*, *Comm. Math. Phys.* **185** (1997) 411 [hep-th/9608169].

[6] L. Baulieu, H. Kanno and I. Singer, *Special Quantum Field Theories in Eight and Other Dimensions*, *Talk given at APCTP Winter School on Dualities in String Theory, (Sokcho, Korea), February 24-28, 1997*, *Comm. Math. Phys.* **194** (1998) 149 [hep-th/9704167];
L. Baulieu, H. Kanno and I. Singer, *Cohomological Yang–Mills Theory in Eight Dimensions*, hep-th/9705127, to appear in *Commun. Math. Phys.*

[7] J. Cheeger and J. Simons, *Differential Characters and Geometric Invariants*, Stony Brook Preprint, 1973, unpublished.

[8] A. Losev, G. Moore, N. Nekrasov and S. Shatashvili, *Four-Dimensional Avatars of Two-Dimensional RCFT*, *Nucl. Phys.* **46** *(Proc. Suppl.)* (1996) 130 [hep-th/9509151];
L. Baulieu, A. Losev and N. Nekrasov, *Chern-Simons and Twisted Supersymmetry in Higher Dimensions*, hep-th/9707174, to appear in *Nucl. Phys. B.*

[9] L. Baulieu and P. West, *Six Dimensional TQFTs and Self-dual Two-Forms.*

[10] N. Seiberg, *Five Dimensional SUSY Field Theories, Non-trivial Fixed Points and String Dynamics*, *Phys. Lett.* **B 388** (1996) 753 [hep-th/9608111].

[11] L. Baulieu, C. Laroche, *On Generalized Self-Duality Equations Towards Supersymmetric Quantum Field Theories Of Forms*, *Mod. Phys. Lett.* **A 13** (1998) 1115 [hep-th/9801014].

[12] E. Bergshoeff, L.A.J. London and P.K. Townsend, *Spacetime Scale-Invariance and the Super p-Brane*, *Class. and Quant. Grav.* **9** (1992) 2545, [hep-th/9206026]

JHEP06(1998)006

[13] L. Baulieu, *Field Antifield Duality, p-Form Gauge Fields and Topological Quantum Field Theories* *Nucl. Phys.* **B 478** (1996) 431 [hep-th/9512026]

[14] see for example L. Castellani, R. D'Auria and P. Fre, *Supergravity and Superstrings, World Scientific 1991*;
M. Duff and B. Nilsson, *Kaluza-Klein Supergravity*, *Phys. Rep.* **130** (1986) 1.